Reasoning across time and the syntacticization of semantics

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§ 1. Introduction.

1. Offer an alternative to **model-theoretic** semantics that has its roots (more) in **proof-theoretic** semantics:
    (a) Take **validity** as the basic semantic notion;
    (b) for a particular linguistic expression α; from the role it plays in various inferences, inductively determine it **inferential properties**;
    (c) understand those properties as **presuppositions**, i.e., **axioms**, for a particular speaker in a particular discourse in which various other inferences are derived from;
    (d) show that the **theory** over such axiom set can double as a model in the sense traditional model-theoretic semanticists understand a model.
2. Explore these ideas in the temporal domain as they relate to the expressions *by, not_before, before* and *after*.

§ 2. Model-theoretic semantics.

The art of interpreting statements presupposes a strict separation between "language" and the (mathematical) "universe" of entities.

van Dalen (2004)

1. By 'model-theoretic semantics' I understand the (semantic) meaning of a linguistic expression α to be explained in terms of **models**.
2. A model $\mathbb{M}$ is a sequence $(A, R_1, \ldots, R_n, F_1, \ldots, F_m, \{c_i \mid i \in I\})$, where A is a (non-empty) **set of entities**, $R_1, \ldots, R_n$ are **relations**, and $F_1, \ldots F_m$ are **functions**, and c_i are elements of A (**constants**).
3. $[\![\cdot]\!]^{\mathbb{M}}$ is a function having the set of linguistic expressions ('language') as its domain and the 'mathematical "universe" of entities' as its codomain:

$$[\![\alpha]\!]^{\mathbb{M}} = \ldots \tag{2.0.1}$$

4. In (2.0.1), the meaning of α, '. . . ', would be an entity, relation or function per $\mathbb{M}$.
5. Sub-sentential meanings β and γ combine in such a way that a (declarative) statement is interpreted as being either (the) true or (the) false.

> *Semantics with no treatment of truth conditions is not semantics.*
>
> Lewis (1970)

$$[\![\beta]\!]^{\mathbb{M}}([\![\gamma]\!]^{\mathbb{M}}) \in \{\top, \bot\} \tag{2.0.2}$$

$$[\![\beta]\!]^{\mathbb{M}} \qquad [\![\gamma]\!]^{\mathbb{M}}$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

6. The operative relation for the semanticist is that of **entailment** (or **logical consequence**):

$$\Gamma \models \varphi \Leftrightarrow \forall \psi \in \Gamma(\mathbb{M} \models \psi \Rightarrow \mathbb{M} \models \varphi) \tag{2.0.3}$$

as it serves as a guide so-to-speak in the pursuit of writing the truth (or denotata-) conditions of an expression α.

7. The notion of 'entailment' is derived from the notion of 'truth'; itself derived from the notion of 'reference'; itself derived from ontological assumptions laid out in $\mathbb{M}$.
8. Thus we get the slogan 'meaning as reference'.

Wyman's overpopulated universe is in many ways unlovely. It offends the aesthetic sense of us who have a taste for desert landscapes.

Quine (1953c)

§ 2.0.1. Doing semantics without doing semantics (lessons from a first-order completeness proof).

. . . we have to construct a model and the only thing we have is our consistent theory. This construction is a kind of Baron von Münchhausen trick; we have to pull ourselves (or rather, a model) out of the quicksand of syntax and proof rules.

van Dalen (2004)

1. A **theory** T is a set of sentences such that if φ is derivable from T than φ is an element of T. (A theory is closed under derivability.)

2. A set Γ such that

$$T = \{\varphi \mid \Gamma \vdash \varphi\} \tag{2.0.4}$$

   is called an **axiom set** of the theory T, where the elements of Γ are referred to as axioms.

3. Assume a first-order language $\mathcal{L}$ with equality such that

$$\frac{}{x = x}\, I_1 \qquad \frac{x = y}{y = x}\, I_2 \qquad \frac{x = y \qquad y = z}{x = z}\, I_3 \tag{2.0.5}$$

   $I_1 - I_3$ are among the rules of the (relevant) proof calculus.

4. Let

$$\Gamma = \left\{ \begin{array}{ll} a = b, & b = c, \\ d = e, & e = f \end{array} \right\} \tag{2.0.6}$$

   be an axiom set.

5. So,

$$T = \left\{ \begin{array}{ccc} a = b, & b = c, & a = c, \\ b = a, & c = b, & c = a, \\ a = a, & b = b, & c = c, \\ d = e, & e = f, & d = f, \\ e = d, & f = e, & f = d, \\ d = d, & e = e, & f = f, \\ \vdots & \vdots & \vdots \end{array} \right\} \tag{2.0.7}$$

is a theory over Γ.

6. Proof rules $I_1 - I_3$ guarantee that, in closing under deduction, structure is imposed upon T. More precisely,

$$(T, =) \tag{2.0.8}$$

is an **equivalence relation**, as '=' is reflexive, symmetric, and transitive:

$$\left\{ [a]_= = [b]_= = [c]_= = \left\{ \begin{array}{ccc} a, & b, & c, \\ \vdots & \vdots & \vdots \end{array} \right\}, [d]_= = [e]_= = [f]_= = \left\{ \begin{array}{ccc} d, & e, & f, \\ \vdots & \vdots & \vdots \end{array} \right\} \right\} \tag{2.0.9}$$

7. From the syntax and proof rules of $\mathcal{L}$, and without any ontological assumptions or appeal to models save for the existence of syntactic expressions themselves, we are able to derive a structure that accords with standard (philosophical) views of equality (Quine 1953b).

§ 3. (Some) temporal expressions.

1. In this section, explore the 'semantics' of the temporal expressions below:

$$\Gamma = \left\{ \begin{array}{cc} \textit{by}, & \textit{not\_before}, \\ \textit{before}, & \textit{after} \end{array} \right\} \tag{3.0.10}$$

2. By investigating the inferences licensed by a particular expression, various properties of that expression can be inductively determined.

§ 3.1. Inferential properties.

1. The expression *by* is **reflexive**:

$$\forall R(\mathsf{Reflexive}(R) \leftrightarrow \forall x(R(x,x))) \tag{3.1.1}$$

(1) ∴ John left by the time he left

2. *By* is **anti-symmetric**:

$$\forall R(\mathsf{Anti\_Symmetric}(R) \leftrightarrow \forall x,y(R(x,y) \wedge R(y,x) \rightarrow x = y)) \tag{3.1.2}$$

(2) a. John left by the time he ate
b. John ate by the time he left
c. ∴ John left at the same time he ate

3. *By* is **transitive**:

$$\forall R(\mathsf{Transitive}(R) \leftrightarrow \forall x,y,z(R(x,y) \wedge R(y,z) \rightarrow R(x,z) \tag{3.1.3}$$

(3) a. John ate by the time he brushed his teeth
b. John brushed his teeth by the time he left
c. ∴ John ate by the time he left

4. *By* is a **total order**:

$$\forall R(\mathsf{Total}(R) \leftrightarrow \forall x,y(R(x,y) \vee R(y,x))) \tag{3.1.4}$$

(4) a. ∴ Either John left by the time he ate or he ate by the time he left
b. ∴ Either John brushed his teeth by the time he left or he left by the time he brushed his teeth
c. . . .

5. Speakers who assent to the validity of (1) – (4) (ostensibly) **presuppose** the meta-semantic statements in (3.1.5), where 'presuppose' simply amounts to the adoption of Γ as an axiom set (and a commitment to its corresponding theory T) by a particular speaker for the purposes of a particular discourse (or multiple discourses):

$$\{\mathsf{Reflexive}(by), \mathsf{Anti\_Symmetric}(by), \mathsf{Transitive}(by), \mathsf{Total}(by)\} \tag{3.1.5}$$

§ 3.2. A definitional extension.

1. We can define the expression *not_before* in terms of *by*:

(5) a. John left by the time he ate
b. ∴ John ate not before he left

$$\forall x, y(\mathit{not\_before}(x, y) \leftrightarrow \mathit{by}(y, x)) \tag{3.2.1}$$

2. The expression *before* can be defined in terms of *by*:

(6) a. John left by the time he ate
b. John didn't leave at the same time he ate
c. ∴ John left before he ate

$$\forall x, y(\mathit{before}(x, y) \leftrightarrow \mathit{by}(x, y) \wedge x \neq y) \tag{3.2.2}$$

3. The expression *after* can be defined in terms of *before*, itself defined of in terms of *by*:

(7) a. John left before he ate
b. ∴ John ate after he left

$$\forall x, y(\mathit{after}(x, y) \leftrightarrow \mathit{before}(y, x)) \tag{3.2.3}$$

4. Speakers who assent to the validity of (1) – (4) and (5) – (7) (ostensibly) **presuppose** the meta-semantic statements in (3.2.4):

$$(3.1.5) \cup \{(3.2.1), (3.2.2), (3.2.3)\} \tag{3.2.4}$$

and are committed to the square of opposition laid out below.

§ 3.2.1. A (temporal) square of opposition.

1. Much like the quantifiers *every, no, not all* and *some* from the (Aristotelean) square of opposition, the temporal expressions *by, not_before, before* and *after* do as well:
   (a) **Contraries** like *after* and *before* cannot both be 'true' at the same time;
   (b) *Not_before* and *by* are **sub-contraries** because they can both be true, but they cannot both be false;
   (c) *Before* is a **sub-altern** of *by*, for example, because the former implies the latter;
   (d) For **contradictories** like *after* and *by*, one must be 'true' while the other 'false'.
2. The logic laid out here is essentially a sub-fragment of Allen's (1983).

$$\begin{tikzcd} \mathit{after}(x,y) \arrow[d, Rightarrow] \arrow[dr, leftrightarrow] & \mathit{before}(x,y) \arrow[d, Rightarrow] \arrow[dl, leftrightarrow] \\ \mathit{not\_before}(x,y) & \mathit{by}(x,y) \end{tikzcd}$$

Figure 1: Temporal square of opposition

§ 3.3. Deriving a temporal structure from language itself.

1. Let
   (a) S_1 be 'John left'
   (b) S_2 be 'John ate breakfast'
   (c) S_3 be 'John brushed his teeth'
   (d) S_4 be 'John woke up'

2. Let

$$\Gamma = (3.2.4) \cup \left\{ \; by(S_1, S_2), \quad by(S_2, S_3), \quad by(S_3, S_4) \; \right\}$$

   be an axiom set.

3. In closing under deduction, structure is imposed upon the resultant theory T. More precisely,

$$(T, by) \tag{3.3.1}$$

   is a (weak) **linear order**, as *by* is reflexive, anti-symmetric, transitive, and total:

$$S_1 \;\; by \; S_2 \;\; by \; S_3 \;\; by \; S_4 \tag{3.3.2}$$

4. From the syntax and proof rules of $\mathcal{L}$, and without assuming that S_i **refers** to or **denotes** a moment (or a set of moments) linearly ordered with respect to the denotation of S_j, for example, we are able to derive a structure isomorphic to standard models of time (Krifka 1998) that involves only the terms of the language itself.

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