§ 1. Introduction.

1. Offer an alternative to model-theoretic semantics that has its roots (more) in proof-theoretic semantics:
   (a) Take validity as the basic semantic notion;
   (b) for a particular linguistic expression $\alpha$; from the role it plays in various inferences, inductively determine it inferential properties;
   (c) understand those properties as presuppositions, i.e., axioms, for a particular speaker in a particular discourse in which various other inferences are derived from;
   (d) show that the theory over such axiom set can double as a model in the sense traditional model-theoretic semanticists understand a model.

2. Explore these ideas in the temporal domain as they relate to the expressions by, not before, before and after.


The art of interpreting statements presupposes a strict separation between “language” and the (mathematical) “universe” of entities.
1. By ‘model-theoretic semantics’ I understand the (semantic) meaning of a linguistic expression $\alpha$ to be explained in terms of **models**.

2. A model $M$ is a sequence $(A, R_1, \ldots, R_n, F_1, \ldots, F_m, \{c_i \mid i \in I\})$, where $A$ is a (non-empty) set of entities, $R_1, \ldots, R_n$ are relations, and $F_1, \ldots, F_m$ are functions, and $c_i$ are elements of $A$ (constants).

3. $[\cdot]^M$ is a function having the set of linguistic expressions (‘language’) as its domain and the ‘mathematical “universe” of entities’ as its codomain:

$$[\alpha]^M = \ldots$$

(2.0.1)

4. In (2.0.1), the meaning of $\alpha$, ‘…’, would be an entity, relation or function per $M$.

5. Sub-sentential meanings $\beta$ and $\gamma$ combine in such a way that a (declarative) statement is interpreted as being either (the) true or (the) false.

**Semantics with no treatment of truth conditions is not semantics.**

Lewis (1970)

$$[\beta]^M([\gamma]^M) \in \{\top, \bot\}$$

(2.0.2)

6. The operative relation for the semanticist is that of entailment (or logical consequence):

$$\Gamma \models \varphi \iff \forall \psi \in \Gamma(M \models \psi \Rightarrow M \models \varphi)$$

(2.0.3)

as it serves as a guide so-to-speak in the pursuit of writing the truth (or denotata-) conditions of an expression $\alpha$.

7. The notion of ‘entailment’ is derived from the notion of ‘truth’; itself derived from the notion of ‘reference’; itself derived from ontological assumptions laid out in $M$.

8. Thus we get the slogan ‘meaning as reference’.

van Dalen (2004)
Wyman’s overpopulated universe is in many ways unlovely. It offends the aesthetic sense of us who have a taste for desert landscapes.

Quine (1953c)

§ 2.0.1. Doing semantics without doing semantics (lessons from a first-order completeness proof).

...we have to construct a model and the only thing we have is our consistent theory. This construction is a kind of Baron von Münchhausen trick; we have to pull ourselves (or rather, a model) out of the quicksand of syntax and proof rules.

van Dalen (2004)

1. A theory $T$ is a set of sentences such that if $\varphi$ is derivable from $T$ than $\varphi$ is an element of $T$. (A theory is closed under derivability.)

2. A set $\Gamma$ such that

$$T = \{ \varphi \mid \Gamma \vdash \varphi \}$$

(2.0.4)

is called an **axiom set** of the theory $T$, where the elements of $\Gamma$ are referred to as axioms.

3. Assume a first-order language $\mathcal{L}$ with equality such that

$$\frac{x = x}{x = x} \quad \frac{x = y}{y = x} \quad \frac{x = y}{y = z} \quad \frac{y = z}{x = z} \quad I_1 \quad I_2 \quad I_3$$

(2.0.5)

$I_1 - I_3$ are among the rules of the (relevant) proof calculus.

4. Let

$$\Gamma = \left\{ \begin{array}{l} a = b, \\ b = c, \\ d = e, \\ e = f \end{array} \right\}$$

(2.0.6)

be an axiom set.
5. So,

\[ T = \begin{cases} 
  a = b, & b = c, & a = c, \\
  b = a, & c = b, & c = a, \\
  a = a, & b = b, & c = c, \\
  d = e, & e = f, & d = f, \\
  e = d, & f = e, & f = d, \\
  d = d, & e = e, & f = f, \\
  \vdots & \vdots & \vdots 
\end{cases} \]  \tag{2.0.7}

is a theory over \( \Gamma \).

6. Proof rules \( I_1 - I_3 \) guarantee that, in closing under deduction, structure is imposed upon \( T \). More precisely,

\[ (T, =) \]  \tag{2.0.8}

is an \textbf{equivalence relation}, as \( '=' \) is reflexive, symmetric, and transitive:

\[ \left\{ \begin{array}{ccc}
  a = b, & b = c, & a = c, \\
  b = a, & c = b, & c = a, \\
  a = a, & b = b, & c = c, \\
  d = e, & e = f, & d = f, \\
  e = d, & f = e, & f = d, \\
  d = d, & e = e, & f = f, \\
  \vdots & \vdots & \vdots 
\end{array} \right\} \]  \tag{2.0.9}

7. From the syntax and proof rules of \( \mathcal{L} \), and without any ontological assumptions or appeal to models save for the existence of syntactic expressions themselves, we are able to derive a structure that accords with standard (philosophical) views of equality (Quine 1953b).

\section*{§ 3. (Some) temporal expressions.}

1. In this section, explore the ‘semantics’ of the temporal expressions below:

\[ \Gamma = \left\{ \begin{array}{cc}
  \text{by,} & \text{not\_before,} \\
  \text{before,} & \text{after} 
\end{array} \right\} \]  \tag{3.0.10}

2. By investigating the inferences licensed by a particular expression, various properties of that expression can be inductively determined.
§ 3.1. Inferential properties.

1. The expression by is reflexive:

\[
\forall R(\text{Reflexive}(R) \leftrightarrow \forall x(R(x,x))) \quad (3.1.1)
\]

(1) \therefore John left by the time he left

2. By is anti-symmetric:

\[
\forall R(\text{Anti-Symmetric}(R) \leftrightarrow \forall x,y(R(x,y) \land R(y,x) \to x = y)) \quad (3.1.2)
\]

(2) a. John left by the time he ate
b. John ate by the time he left
c. \therefore John left at the same time he ate

3. By is transitive:

\[
\forall R(\text{Transitive}(R) \leftrightarrow \forall x,y,z(R(x,y) \land R(y,z) \to R(x,z))) \quad (3.1.3)
\]

(3) a. John ate by the time he brushed his teeth
b. John brushed his teeth by the time he left
c. \therefore John ate by the time he left

4. By is a total order:

\[
\forall R(\text{Total}(R) \leftrightarrow \forall x,y(R(x,y) \lor R(y,x))) \quad (3.1.4)
\]

(4) a. \therefore Either John left by the time he ate or he ate by the time he left
b. \therefore Either John brushed his teeth by the time he left or he left by the time he brushed his teeth
c. \ldots

5. Speakers who assent to the validity of (1) – (4) (ostensibly) presuppose the meta-semantic statements in (3.1.5), where ‘presuppose’ simply amounts to the adoption of \( \Gamma \) as an axiom set (and a commitment to its corresponding theory \( T \)) by a particular speaker for the purposes of a particular discourse (or multiple discourses):

\[
\{\text{Reflexive(by)}, \text{Anti-Symmetric(by)}, \text{Transitive(by)}, \text{Total(by)}\} \quad (3.1.5)
\]
§ 3.2. A definitional extension.

1. We can define the expression *not_before* in terms of *by*:

   (5) a. John left by the time he ate  
       b. \(\therefore\) John ate not before he left  

   \[ \forall x, y (\text{not_before}(x, y) \leftrightarrow \text{by}(y, x)) \]  

(3.2.1)

2. The expression *before* can be defined in terms of *by*:

   (6) a. John left by the time he ate  
       b. John didn’t leave at the same time he ate  
       c. \(\therefore\) John left before he ate  

   \[ \forall x, y (\text{before}(x, y) \leftrightarrow \text{by}(x, y) \land x \neq y) \]  

(3.2.2)

3. The expression *after* can be defined in terms of *before*, itself defined of in terms of *by*:

   (7) a. John left before he ate  
       b. \(\therefore\) John ate after he left  

   \[ \forall x, y (\text{after}(x, y) \leftrightarrow \text{before}(y, x)) \]  

(3.2.3)

4. Speakers who assent to the validity of (1) – (4) and (5) – (7) (ostensibly) presuppose the meta-semantic statements in (3.2.4):

   \[ \text{(3.1.5)} \cup \{\text{(3.2.1)}, \text{(3.2.2)}, \text{(3.2.3)}\} \]  

(3.2.4)

and are committed to the square of opposition laid out below.

§ 3.2.1. A (temporal) square of opposition.

1. Much like the quantifiers *every, no, not all* and *some* from the (Aristotelean) square of opposition, the temporal expressions *by, not_before, before* and *after* do as well:

   (a) **Contraries** like *after* and *before* cannot both be ‘true’ at the same time;  
       (b) *Not_before* and *by* are **sub-contraries** because they can both be true, but they cannot both be false;  
       (c) *Before* is a **sub-altern** of *by*, for example, because the former implies the latter;  
       (d) For **contradictories** like *after* and *by*, one must be ‘true’ while the other ‘false’.

2. The logic laid out here is essentially a sub-fragment of Allen’s (1983).
§ 3.3. Deriving a temporal structure from language itself.

1. Let
   
   (a) $S_1$ be ‘John left’
   (b) $S_2$ be ‘John ate breakfast’
   (c) $S_3$ be ‘John brushed his teeth’
   (d) $S_4$ be ‘John woke up’

2. Let

   $$\Gamma = (3.2.4) \cup \{ \text{by}(S_1, S_2), \text{by}(S_2, S_3), \text{by}(S_3, S_4) \}$$

   be an axiom set.

3. In closing under deduction, structure is imposed upon the resultant theory $T$. More precisely,

   $$(T, \text{by}) \quad (3.3.1)$$

   is a (weak) **linear order**, as $by$ is reflexive, anti-symmetric, transitive, and total:

   $$S_1 \text{ by } S_2 \text{ by } S_3 \text{ by } S_4 \quad (3.3.2)$$

4. From the syntax and proof rules of $\mathcal{L}$, and without assuming that $S_i$ **refers** to or **denotes** a moment (or a set of moments) linearly ordered with respect to the denotation of $S_j$, for example, we are able to derive a structure isomorphic to standard models of time (Krifka 1998) that involves only the terms of the language itself.
References.


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